This unique ECP design vividly demonstrates the need for and effectiveness of closed loop control. It is not the conventional rod-on-cart inverted pendulum, but rather, it steers a horizontal balancing rod in the presence of gravity to control the vertical pendulum rod. As detailed in the manual, the plant has both right half plane poles and zeros as well as kinematic and gravitationally coupled nonlinearities. By adjusting mass properties, these roots may be varied to make the control problem range from being relatively simple to theoretically impossible! The system includes removable and adjustable moment arm counterweights on the vertical and horizontal rods for quick adjustment of the plant dynamics. It features linear and rotary ball bearings at the joints for low friction and consistent dynamic properties.

### Plant Model

**Dynamic Equations**

- **Exact**
  \[
  m_2\ddot{x} + m_1\ddot{\theta} - m_1x\dot{\theta}^2 - mg\sin(\theta) = F_0 \\
  m_1\dot{x}\ddot{\theta} + m_1x\dot{\theta}^2 + 2m_1x\dot{x}\dot{\theta} - (m_1l_0^2 + m_2c_1^2)g\sin(\theta) - m_1g(x\cos(\theta) - 0 = 0 \\
  J_0(\dot{\theta}) = J_1 + m_1(l_0^2 + x^2) + J_1m_2c_1^2
  \]

- **Linearized Time Domain**
  \[
  m_1\ddot{x} + m_1\dot{x}\ddot{\theta} - mg\theta = F_0 \\
  m_1\dot{x}\ddot{\theta} - (m_1l_0 + m_1c_1)\ddot{\theta} - m_2g(\theta - 0 = 0 \\
  J_0' = J_0|_{\theta = 0}
  \]

- **S-Domain**
  \[
  \frac{\theta(s)}{F_0} = \frac{s^2}{J_0 - m_1c_1^2s^4 + (m_2l_0 - m_1l_0)g s^2 - mg^2}
  \]

### Characteristics

- Nonlinearities in kinematic constraints and coordinate dependent mass properties.
- Linearization about \( x = 0 \), \( \theta = 0 \) shown to be valid for many control schemes.
- One RHP, 2 oscillatory poles.
- Nonmin phase (RHP zero).
- Attainable bandwidth bounded from above and below by RHP roots.